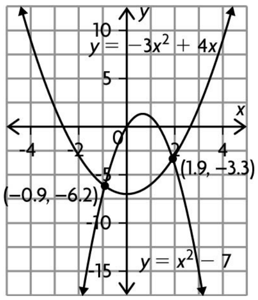
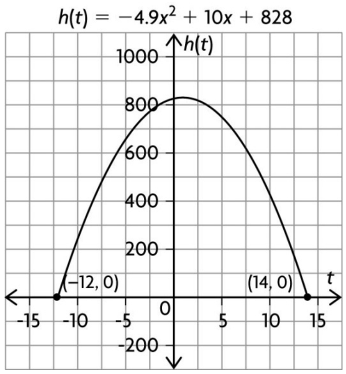
6. The x-intercepts are –0.9 and 1.9.



7. Here is the diagram.



The positive intercept is $t = 14$.
The freefall lasts 14 s.

8. $\frac{-4+2}{2} = -1$

So the axis of symmetry is $x = -1$.

9. The x-intercepts are –4 and 2.
So the equation is $y = a(x + 4)(x - 2)$.
The y-intercept is –2.

$-2 = a(0 + 4)(0 - 2)$
$-2 = -8a$
$0.25 = a$
$y = 0.25(x + 4)(x - 2)$
$y = 0.25x^2 + 0.5x - 2$

10. a) The equation is $y = a(x + 6)(x - 12)$.
The y-intercept is –36.

$-36 = a(0 + 6)(0 - 12)$
$-36 = -72a$
$0.5 = a$
$y = 0.5(x + 6)(x - 12)$
$y = 0.5x^2 - 3x - 36$

b) The axis of symmetry is $x = \frac{-6+12}{2} = 3$.

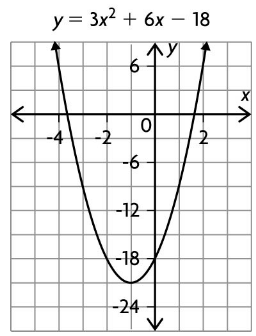
$f(3) = 0.5(3 + 6)(3 - 12)$
$f(3) = 0.5(9)(-9)$
$f(3) = -40.5$
The vertex is (3, –40.5).

c) Domain and range: $\{(x, y) \mid x \in R, y \geq -40.5, y \in R\}$

11. $y = 3x^2 + 6x - 18$
$y = 3x(x + 2) - 18$
$3x = 0 \qquad x + 2 = 0$
$x = 0 \qquad x = -2$
So, (0, –18) and (–2, –18) are points on the parabola.

$\frac{0+(-2)}{2} = -1$

$y = 3(-1)^2 + 6(-1) - 18$
$y = -21$
So, the vertex is at (–1, –21).



12. The revenue is the price times the number of bicycles. Let the number of decreases in price be x and the revenue be y.

$y = (39 - 1.5x)(63 + 7x)$
$y = -10.5x^2 + 178.5x + 2457$
$y = -10.5(x^2 - 17x - 234)$
Factoring:
$y = -10.5(x + 9)(x - 26)$

The vertex is at $x = \frac{-9+26}{2}$ or 8.5.

The maximum revenue would be achieved at a price of \$(39 – 1.5(8.5)) = \$26.25.

Lesson 7.5: Solving Quadratic Equations by Factoring, page 405

1. a) $x^2 - 11x + 28 = (x - 4)(x - 7)$
$x - 4 = 0$
$x = 4$
or
$x - 7 = 0$
$x = 7$
The roots are 4 and 7.

b) $x^2 - 7x - 30 = (x - 10)(x + 3)$
$x - 10 = 0$
$x = 10$
or
$x + 3 = 0$
$x = -3$
The roots are -3 and 10.

c) $2y^2 + 11y + 5 = (y + 5)(2y + 1)$
$y + 5 = 0$
$y = -5$
or
$2y + 1 = 0$
$y = -0.5$
The roots are –5 and –0.5.

d) $4t^2 + 7t - 15 = (4t - 5)(t + 3)$
$4t - 5 = 0$
$t = 1.25$
or
$t + 3 = 0$
$t = -3$
The roots are –3 and 1.25.
2. a) $x^2 - 121 = (x - 11)(x + 11)$
$x - 11 = 0$
$x = 11$
or
$x + 11 = 0$
$x = -11$
The roots are –11 and 11.
b) $9r^2 - 100 = (3r - 10)(3r + 10)$
$3r - 10 = 0$
$r = \frac{10}{3}$
or
$3r + 10 = 0$
$r = -\frac{10}{3}$
The roots are $-\frac{10}{3}$ and $\frac{10}{3}$.
c) $x^2 - 15x = x(x - 15)$
$x = 0$
or
$x - 15 = 0$
$x = 15$
The roots are 0 and 15.
d) $3y^2 + 48y = 3y(y + 16)$
$3y = 0$
$y = 0$
or
$y + 16 = 0$
$y = -16$
The roots are –16 and 0.
e) $s^2 - 12s + 36 = (s - 6)(s - 6)$
$s - 6 = 0$
$s = 6$
The root is 6.
f) $16p^2 + 8p + 1 = (4p + 1)(4p + 1)$
$4p + 1 = 0$
$p = -0.25$
The root is –0.25.
g) $-14z^2 + 35z = -7z(2z - 5)$
$-7z = 0$
$z = 0$
or
$2z - 5 = 0$
$z = 2.5$
The roots are 0 and 2.5.
h) $5q^2 - 9q = 5q\left(q - \frac{9}{5}\right)$
$5q = 0$
$q = 0$
or
$q - \frac{9}{5} = 0$
$q = \frac{9}{5}$
The roots are 0 and $\frac{9}{5}$.
3. a) $x^2 - 9x - 70 = (x - 14)(x + 5)$
$x - 14 = 0$
$x = 14$
or
$x + 5 = 0$
$x = -5$
The roots are –5 and 14.
$(-5)^2 - 9(-5) - 70 = 25 + 45 - 70$
$(-5)^2 - 9(-5) - 70 = 0$
$(14)^2 - 9(14) - 70 = 196 - 126 - 70$
$(14)^2 - 9(14) - 70 = 0$
b) $x^2 + 19x + 48 = (x + 16)(x + 3)$
$x + 16 = 0$
$x = -16$
or
$x + 3 = 0$
$x = -3$
The roots are –16 and –3.
$(-16)^2 + 19(-16) + 48 = 256 - 304 + 48$
$(-16)^2 + 19(-16) + 48 = 0$
$(-3)^2 + 19(-3) + 48 = 9 - 57 + 48$
$(-3)^2 + 19(-3) + 48 = 0$
c) $3a^2 + 11a - 4 = (3a - 1)(a + 4)$
$3a - 1 = 0$
$a = \frac{1}{3}$
or
$a + 4 = 0$
$a = -4$
The roots are –4 and $\frac{1}{3}$.
$3(-4)^2 + 11(-4) - 4 = 48 - 44 - 4$
$3(-4)^2 + 11(-4) - 4 = 0$
$3\left(\frac{1}{3}\right)^2 + 11\left(\frac{1}{3}\right) - 4 = \frac{1}{3} + \frac{11}{3} - 4$
$3\left(\frac{1}{3}\right)^2 + 11\left(\frac{1}{3}\right) - 4 = \frac{12}{3} - 4$
$3\left(\frac{1}{3}\right)^2 + 11\left(\frac{1}{3}\right) - 4 = 0$

d) $6t^2 - 7t - 20 = (2t - 5)(3t + 4)$

$2t - 5 = 0$

$t = \frac{5}{2}$

or

$3t + 4 = 0$

$t = -\frac{4}{3}$

The roots are $-\frac{4}{3}$ and $\frac{5}{2}$.

$$6\left(-\frac{4}{3}\right)^2 - 7\left(-\frac{4}{3}\right) - 20 = \frac{32}{3} + \frac{38}{3} - 20$$

$$6\left(-\frac{4}{3}\right)^2 - 7\left(-\frac{4}{3}\right) - 20 = \frac{60}{3} - 20$$

$$6\left(-\frac{4}{3}\right)^2 - 7\left(-\frac{4}{3}\right) - 20 = 0$$

$$6\left(\frac{5}{2}\right)^2 - 7\left(\frac{5}{2}\right) - 20 = \frac{75}{2} - \frac{35}{2} - 20$$

$$6\left(\frac{5}{2}\right)^2 - 7\left(\frac{5}{2}\right) - 20 = \frac{40}{2} - 20$$

$$6\left(\frac{5}{2}\right)^2 - 7\left(\frac{5}{2}\right) - 20 = 0$$

4. a) $2x^2 + 5x - 12 = (2x - 3)(x + 4)$

$2x - 3 = 0$

$x = \frac{3}{2}$

or

$x + 4 = 0$

$x = -4$

The roots are –4 and $\frac{3}{2}$.

b) $4x^2 + 9x - 9 = (4x - 3)(x + 3)$

$4x - 3 = 0$

$x = \frac{3}{4}$

or

$x + 3 = 0$

$x = 3$

The roots are –3 and $\frac{3}{4}$.

c) $49d^2 + 42d + 9 = (7d + 3)(7d + 3)$

$7d + 3 = 0$

$d = -\frac{3}{7}$

The root is $-\frac{3}{7}$.

d) $81g^2 - 169 = (9g - 13)(9g + 13)$

$9g - 13 = 0$

$g = \frac{13}{9}$

or

$9g + 13 = 0$

$g = -\frac{13}{9}$

The roots are $-\frac{13}{9}$ and $\frac{13}{9}$.

5. a) $20x^2 - 21x - 27 = (5x - 9)(4x + 3)$

$5x - 9 = 0$

$x = 1.8$

or

$4x + 3 = 0$

$x = -0.75$

The roots are –0.75 and 1.8.

b) e.g., Geeta may have had the wrong signs between the terms within each factor.

6. a) $5u^2 - 10u - 315 = 5(u^2 - 2u - 63)$

$5u^2 - 10u - 315 = 5(u - 9)(u + 7)$

$5(u - 9) = 0$

$u = 9$

or

$u + 7 = 0$

$u = -7$

The roots are –7 and 9.

b) $0.25x^2 + 1.5x + 2 = 0.25(x^2 + 6x + 8)$

$0.25x^2 + 1.5x + 2 = 0.25(x + 2)(x + 4)$

$0.25(x + 2) = 0$

$x = -2$

or

$x + 4 = 0$

$x = -4$

The roots are –4 and –2.

c) $1.4y^2 + 5.6y - 16.8 = 1.4(y^2 + 4y - 12)$

$1.4y^2 + 5.6y - 16.8 = 1.4(y + 6)(y - 2)$

$y + 6 = 0$

$y = -6$

or

$y - 2 = 0$

$y = 2$

The roots are –6 and 2.

d) $\frac{1}{2}k^2 + 5k + 12.5 = \frac{1}{2}(k^2 + 10k + 25)$

$\frac{1}{2}k^2 + 5k + 12.5 = \frac{1}{2}(k + 5)^2$

$k + 5 = 0$

$k = -5$

The root is –5.

7. e.g., $(x + 5)(x + 12) = 0$

$x^2 + 17x + 60 = 0$

8. $-40(x-5)^2 + 25000 = 21000$

$-40(x-5)^2 = -4000$

$(x-5)^2 = 100$

$x-5 = 10$ or $x-5 = -10$

$x = 15$ or $x = -5$

Fifteen increases of 10¢ would make the ticket price $3.50.

Five decreases of 10¢ would make the ticket price $1.50.

9. $20x^2 - 37x + 8 = 0$

The possible factor pairs for 20 are 1, 20; 2, 10; and 4, 5.

The possible factor pairs for 8 are 1, 8 and 2, 4.

The middle term is odd, so 5 is the only number that will give an odd sum.

The factor pair for 20 must be 4, 5.

$4(8) + 5(1) = 32 + 5$ or 37

The factor pair for 8 must be 1, 8.

$20x^2 - 37x + 8 = (5x-8)(4x-1)$

$5x - 8 = 0$

$x = \frac{8}{5}$

or

$4x - 1 = 0$

$x = \frac{1}{4}$

The roots are $\frac{1}{4}$ and $\frac{8}{5}$.

10. Going from the second line to the third line, 100 divided by 5 is 20, not 25. Also, in the final step, it is possible that the final result could be positive or negative. Therefore, the roots are $-\sqrt{20}$ and $\sqrt{20}$.

11. The first line was incorrectly factored.

$4r^2 - 9r = 0$

$r(4r-9) = 0$

$r = 0$

or

$4r - 9 = 0$

$4r = 9$

$r = \frac{9}{4}$

The roots are 0 and $\frac{9}{4}$.

12. a) e.g., $y = 8(x-0.5)(x+0.75)$

$y = 8x^2 + 2x - 3$

b) e.g., No, we had different functions.

c) e.g., Multiply by 2 and by –3.

$y = 16x^2 + 4x - 6$, $y = -24x^2 - 6x + 9$

13. a) $0 = -0.25n^2 + 6n - 27$

$0 = -0.25(n^2 - 24n + 108)$

$0 = -0.25(n-6)(n-18)$

$n - 6 = 0$

$n = 6$

or

$n - 18 = 0$

$n = 18$

She must sell 600 or 1800 posters to break even.

b) $5 = -0.25n^2 + 6n - 27$

$0 = -0.25n^2 + 6n - 32$

$0 = -0.25(n^2 - 24n + 128)$

$0 = -0.25(n-8)(n-16)$

$n - 8 = 0$

$n = 8$

or

$n - 16 = 0$

$n = 16$

She must sell 800 or 1600 posters to make $5000.

c) $9 = -0.25n^2 + 6n - 27$

$0 = -0.25n^2 + 6n - 36$

$0 = -0.25(n^2 - 24n + 144)$

$0 = -0.25(n-12)(n-12)$

$n - 12 = 0$

$n = 12$

She must sell 1200 posters to make $9000.

d) D: $n \geq 0$; R: $-27 \leq P \leq 9$

If Sanela sells 0 posters, she will incur a loss of $27 000; her maximum profit is $9000.

14. a) $0 = -25t^2 - 5t + 1260$

$0 = -5(5t^2 + t - 252)$

Factor pairs of 5: 5, 1

Factor pairs of 252: 252, 1; 3, 84; 6, 42, 12, 21; 36, 7; …

5(7) = 35 and 1(36) = 36

$0 = -5(5t + 36)(t - 7)$

$5t + 36 = 0$

$t = -7.2$

or

$t - 7 = 0$

$t = 7$

$t = 7$ or $t = -7.2$

t must be positive.

The rock will hit the water in 7 s.

b) The rock hits the water at 7 s so:

D: $0 \leq t \leq 7$, where t is the time in seconds.

15. a) e.g. $x^2 - 2x - 8 = 0$

b) e.g. $x^2 - 2x - 9 = 0$; I changed the “c” term.

16. a) i) Write the equation in standard form.

ii) Factor fully.

iii) Set each factor with a variable equal to zero (since the product is zero, one factor must be equal to zero).

iv) Solve.

b) When the quadratic equation is factorable, solve by factoring; otherwise, solve by graphing.

17. a) Since the equation is factorable, I can predict that it is a difference of squares.

b) The other root is –6.

c) $\left(\sqrt{a}x + \sqrt{c}\right)\left(\sqrt{a}x - \sqrt{c}\right) = 0$, where $\frac{\sqrt{c}}{\sqrt{a}} = 6$

d) $ax^2 - c = 0$

18. The third side is $60 - n - 2n - 6 = 54 - 3n$.
Solve for n using the Pythagorean theorem.

$$\begin{aligned} n^2 + (54 - 3n)^2 &= (2n + 6)^2 \\ n^2 + 2916 - 324n + 9n^2 &= 4n^2 + 24n + 36 \\ 6n^2 - 348n + 2880 &= 0 \end{aligned}$$

Divide by 6:

$$n^2 - 58n + 480 = 0$$

Factor pairs of 480: 2, 240; 4, 120; 8, 60; 16, 30; 10, 48…

$$\begin{aligned} (n - 10)(n - 48) &= 0 \\ n &= 10 \text{ or } n = 48 \end{aligned}$$

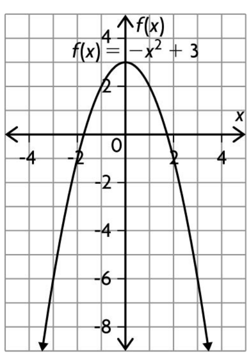
n cannot be 48 because it would make the third side $(54 - 3n)$ negative.

$$\begin{aligned} n &= 10 \\ 2n + 6 &= 26 \\ 54 - 3n &= 24 \end{aligned}$$

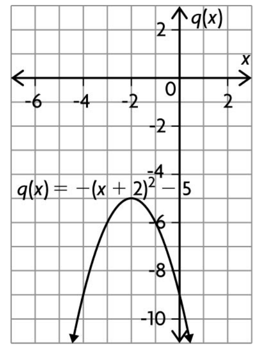
The sides of the triangles are 10 cm, 24 cm, and 26 cm.

Lesson 7.6: Vertex Form of a Quadratic Function, page 417

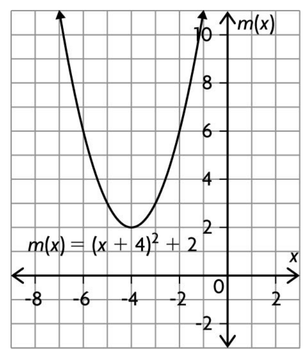
1. a) i) $a > 0$, so the parabola opens upward.
ii) The vertex is (3, 7).
iii) The axis of symmetry is $x = 3$.
b) i) $a < 0$, so the parabola opens downward.
ii) The vertex is (–7, –3).
iii) The axis of symmetry is $x = -7$.
c) i) $a > 0$, so the parabola opens upward.
ii) The vertex is (2, –9).
iii) The axis of symmetry is $x = 2$.
d) i) $a > 0$, so the parabola opens upward.
ii) The vertex is (–1, 10).
iii) The axis of symmetry is $x = -1$.
e) i) $a < 0$, so the parabola opens downward.
ii) The vertex is (0, 5).
iii) The axis of symmetry is $x = 0$.
2. a) $a < 0$, so the parabola opens downward and has a maximum at (0, 3). The maximum is above the x-axis, so it will have two x-intercepts.



b) $a < 0$, so the parabola opens downward and has a maximum at (–2, –5). The maximum is below the x-axis, so it will have no x-intercepts.



c) $a > 0$, so the parabola opens upward and has a minimum at (–4, 2). The minimum is above the x-axis, so it will have no x-intercepts.



d) $a > 0$, so the parabola opens upward and has a minimum at (3, –6). The minimum is below the x-axis, so it will have two x-intercepts.